A Taylor Rule with Monthly Data

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Abstract. The celebrated Taylor rule methodology has established that the decisions made by the Federal Open Market Committee concerning possible changes in short-term interest rates reflected in Federal funds are influenced by deviations from a desired level of inflation and from potential output. A large bibliography has emerged using different methodologies to investigate numerous aspects of the Taylor rule. In this study we use not only time series and econometric modeling, but also neural network methodology with monthly data from 1958 to the end of 2005. We distinguish between sample and out-of-sample sets to train and evaluate the model's effectiveness.

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1. Introduction

The basic purpose of this paper is to forecast monthly Federal funds rates using both econometric and neural network methodologies. A graph indicating the path of the Federal funds rates from 1957 through 2005 is shown in Figure 1.

A number of econometric interest-rate forecasting methodologies have been investigated in the literature. For an account of some of the methodologies that have been applied, see Svensson (2003). Rather than considering every available method, we shall restrict ourselves to the following three approaches:

1. Federal Funds rates are determined solely by past rates.
2. Federal Funds are functions of past influential factors.
3. Federal Funds are functions of past rates as well as influential factors.

Each of these econometric approaches, plus the neural network approach, is discussed in turn.

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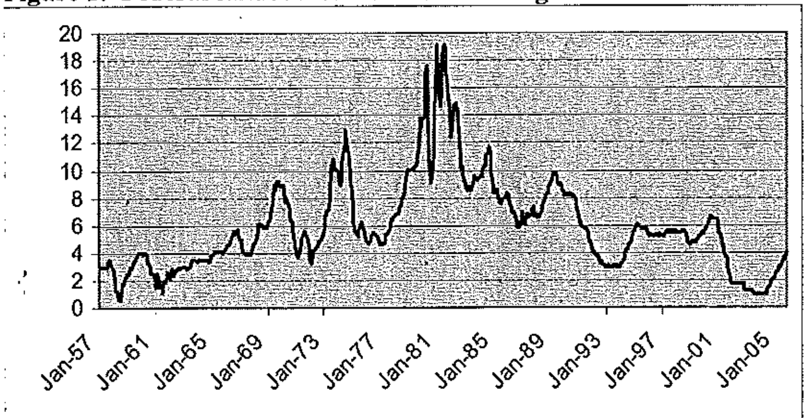
Figure 1: Federal funds rates from 1957 through 2005



2. Federal Funds Are Determined Solely by Past Rates

Much research has been conducted using a continuous-time short-term-rate model specification such as:

$$dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dz \qquad (1)$$

where: r = short-term interest rate
α, β, γ = model coefficients to be determined

σ = standard deviation of the short-term rates
z = Brownian motion

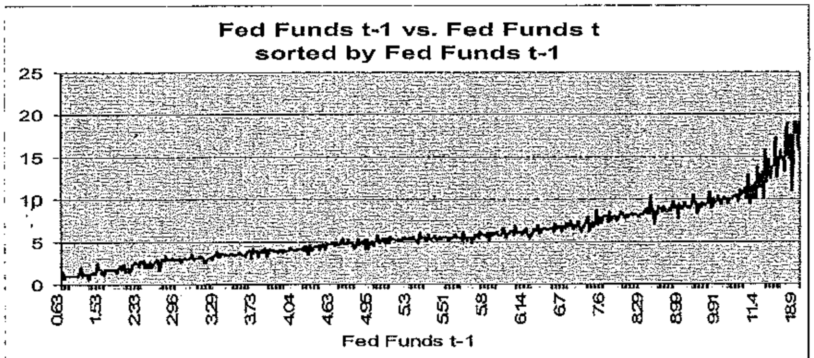
This formulation assumes that movements in interest rates are strictly a function of interest-rate levels and volatility. For investigations of such formulations, see Brenner, Harjes, and Kroner(1996). From (1) a discrete time model can be obtained:

$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t \qquad (2)$$

where: r_t = short-term interest rate at time t
r_{t-1} = short-term interest rate at time t - 1
ε_t = model error term at time
$E(\varepsilon_t) = 0$
α, β = model coefficients to be determined.

Depending on the date range evaluated, the value of β is normally found to be very significant and close to 1. This indicates that interest rates have high serial correlation. Such a result is to be expected since, on average, interest rates are only changed at most monthly by the Federal Reserve Board. In the sections that follow, the model described in (2) will be used as a base model on which to evaluate the effectiveness of other models. Figure 2 show the relationship between Federal funds at time t-1 and time t, sorted by funds at t-1. Notice the close-to-linear relationship for all but the highest values on the graph.

Figure 2: Federal funds at time t-1 vs. at time t



3. Interest Rates Are Functions of Past Influential Factors

The most famous Federal funds model is the one proposal by Taylor (1993). It argues that Federal funds are determined by the Fed's objectives to promote price stability and economic growth. There is both a quarterly and monthly version. We concentrate on the monthly version where:

$$r_t = 2 + p_{t-1} + 1/2\,(p_{t-1} - 2) + 1/2(u_{t-1} - 4) \tag{3}$$

where: r_t = Federal funds rate at t
p_{t-1} = lagged monthly inflation measured by CPI
u_{t-1} = lagged monthly unemployment rate

Note that this equation indicates that the Federal funds rate should be changed 1.5 percent for each 1 percent change in inflation above a target rate of 2 percent. It is felt that such a forceful reaction to inflation tends to drive future inflation to lower value. Judd and Rudebusch (1998) show that when interest rates are not adjusted strongly in reaction to past inflation, the result can be rampant future inflation similar to the inflation exhibited during the era of 1970 – 1978.

The Taylor rule has become the basis for comparison and development of other policy reaction functions. Modifications to the Taylor rule include the addition of other variables as exemplified by Clarida, Galí, Gertler (1998). Other considerations include the addition of expectations of future values of inflation and output, as shown in Orphanides (2001b). We have used the Taylor rule in two formulations. In the first, values for the coefficients have been taken as standard formulations. In the second, we have solved for the coefficients on the each specific subset of the data.

4. Interest Rates Are Functions of Past Rates as Well as Influential Factors

The third approach combines the factors of the Taylor rule with previous interest rates. By combining equations (2) and (3), we obtain the following, using slightly different coefficient symbols:

$$r_t = \alpha + \rho r_{t-1} + \beta(p_{t-1} - 2) + \lambda(u_{t-1} - 4) + \varepsilon_t \tag{4}$$

where: r_t = short-term interest rate at time t

p_{t-1} = inflation rate at time $t-1$

$u_{t-1} - 4$ = excess unemployment

ε_t = model error term at time t

$E(\varepsilon_t) = 0$

$\alpha, \beta, \lambda, \rho$ = model coefficients to be determined

Numerous investigators have evaluated equations of this form using past values of inflation and excess unemployment for various time intervals and various countries including Judd and Rudebusch (1998), and Clarida, Galí, and Gertler (1998).

5. Interest Rates Can Be Determined by a Neural Network Using Past Rates and Influential Factors

A neural network is a non-linear estimator using weighted interconnected nodes to generate a forecast. It is very dependent upon the training data set since it adjusts its weights to optimize performance on this training data, but has the ability to often outperform linear models on complex data sets. The network contains three layers. The input layer has one node corresponding to each input variable. The output layer has one node corresponding to each desired output. In between these, the hidden layer is a set of nodes with no direct variable interpretation, but which serves to mold the form of the inputs to the output. In this test, we shall use the Federal funds, the CPI and the unemployment rate as inputs to forecast the Federal funds rate at the next time period.

6. Our Data

We use monthly data for Federal funds, inflation measured by the CPI and unemployment from January 1957 to December 2005. CPI data is annualized by calculating $\ln(x_t/x_{t-12})*100$ for each month. Percentages are adjusted to whole numbers, for example, 4 percent is used as 4, not .04.

The computations for each model are performed for various subsamples of the set. These subsamples are divided first into two distinct time periods, then into three sets by value of the current Federal funds rate. These five sets include: time prior to Greenspan (1957 through July 1987), since Greenspan (August 1987 through November 2005), the months where the adjusted Federal funds rate was less than 5, between 5 and 10, and greater than 10. For each subsample, a random set of 10 percent of the rows was held out from training and used as the validation set. The models are all compared by looking at their performance on these validation sets. Sizes of each of the model training and validation sets are shown in Table 1.

As explained in prior sections, not all variables are used in all models. The random walk model uses only the current Federal funds number as an independent variable. The Taylor model uses the CPI and the unemployment rate. The econometric and neural network models use current Federal funds, the CPI, and the unemployment rate. The dependent variable for all models is Federal funds for the next month.

Table 1. Data Set Sizes

Data Set	Training	Validation	Total
PreGreenspan	319	36	355
Greenspan	197	22	219
r_{t-1} : 0 to 5	219	24	243
r_{t-1} : 5.01 to 10	243	27	270
r_{t-1} : over 10	55	6	61

7. Model Results on Validation Sets

The mean squared error has been calculated for each of the five models tested over each of the five subsets of data. The training and validation sets are distinct. Results show the lowest error amounts comes from the models using all three of the variables for input. That is, in each subset of data, the lowest error came from either the econometric or neural network model. The random walk model is very close to the lowest error in each subset, but never is the lowest. The two formulations of the Taylor model have significantly greater errors than any of the other three, over all data subsets. The results are shown in Table 2 with the lowest error in bold.

Table 2. Mean Squared Error Comparisons on Validation Sets

Model / Data Set	PreGreenspan	Greenspan	Low	Medium	High
Random Walk	0.676	0.034	0.122	0.271	0.574
Taylor	10.036	8.392	6.651	9.701	16.754
Taylor2	6.793	3.001	0.985	2.221	1.263
Econometric	**0.657**	**0.030**	0.124	**0.262**	0.613
Neural Network	1.121	0.129	**0.104**	0.269	**0.372**

Notice, in the results from Table 2, we see that the neural network is not the best model when the data is split simply by time period. However, when the data is split by type based on current Federal funds level, the neural network outperforms the random walk each time, and in two of the three sets, is best overall.

8. Conclusions

This paper has reviewed three methods for modeling the behavior of Federal funds. They are the standard random walk, an econometric model that relates the Federal funds to fundamental variables including past values of Federal funds and also the neural network approach. Using monthly data from 1958 to 2005 of several important macroeconomic variables, the results show that the econometric modeling performs better than the other approaches when the data are divided into two sets of pre-Greenspan and Greenspan. However, when the data sample is divided into three groups of low, medium and high Federal funds, the neural network approach does best. The main conclusion of our work is that separating the data set into more homogeneous segments makes it possible for the neural network methodology to outperform both the random walk and Taylor approaches.

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