MODELING THE BEHAVIOR OF THE S&P 500 INDEX: A NEURAL NETWORK APPROACH

1. INTRODUCTION

On Black Monday, October 19, 1987, in less than four hours, the US stock markets lost 1.5 trillion dollars -- five times the amount of the US annual budget deficit. Since this dramatic and surprising 23% decline, as measured by the S&P 500 Index, researchers have intensified their efforts to characterize the behavior of stock market prices. Is the behavior of the market predictable? For a long time, it was believed that price changes were, to a large extent, random [Working (1934), Kendall (1953)]. However, the October 1987 stock market crash challenged the then prevailing financial models of a random walk and led to the emergence of a new and competing model of stock price time series. This new approach supports a non-random underlying structure and is labeled chaotic dynamics.

The chaotic dynamics approach is a deterministic model which yields a time series behavior that appears random when in fact such a series is generated by a nonlinear deterministic equation. Sometimes small input changes can produce very divergent outputs, causing a seemingly orderly system to become chaotic. The chaos appears to be random and unpredictable while it is actually following strict mathematical principles. Although deterministic, chaotic dynamics, when graphed, looks like a random walk. Preliminary statistical evidence has not succeeded in rejecting the presence of chaos in the S&P 500 Index series. The possibility that the underlying dynamics of the S&P 500 might follow a model of low order, nonlinear, deterministic chaos, motivates the search for a neural network which can indicate the existence of such a structure.

Neural networks, developed to pattern the way a brain learns, can develop a representation of variables and their relationships without requiring that this relationship be specified in advance by the developer. A neural network uses an abundance of input data that require categorizing and interpreting. It is not necessary to specify an underlying structure, as the network infers the patterns by generalizing from the experience of the inputs. Neural networks are structured layers of neurons, or processing elements, and connections. The layers include an input layer, an output layer, and one or several interior layers called the middle or hidden layers. Each layer is connected to the next with signals traveling in one direction only, from the input layer to the hidden layers to the output layer. Each connection between neurons has a numerical weight, either positive or negative, associated with it which reinforces or inhibits the effect of the neuron on the next layer. The connection weights initially are random and are adjusted as the input data are repeatedly fed through the layers of the network. If a neural network can be constructed which predicts market prices better than 50% of the time, this would imply that the network has discovered an underlying structure in the data. Such a result would be in direct conflict with the random walk model and would support those who believe that a chaotic dynamics structure underlies the market. The appendix provides a technical discussion of these models for the interested readers.

2. RESEARCH PLAN AND SIGNIFICANCE

The project consists of four basic components: (a) bibliographic research and data collection; (b) fitting autoregressive and moving average models to the data to check for statistical evidence of a random walk; (c) construction and training of a neural network; and (d) comparison of the models using three criteria.

The bibliographic search and data collection have been completed. Weekly data have been collected encompassing the past three years on ten variables, including the S&P 500 Index, the three month Treasury Bill interest rate, the thirty year Treasury Bond interest rate, New York Stock Exchange volume, Money Supply as measured by both M1 and M2, Price/Earnings ratio, Gold prices, Crude Oil prices, and the CBOE put/call ratio. The stock market is influenced by expectations of the traders, fundamental measures of economic activity and technical factors such as trading volume. Some representative references that discuss the selection of these variables are Estrella and Hardouvelis (1991), Fama (1990) and Malliaris and Malliaris (1992).

During the second phase, time series models will be fitted on windows of moving subsets of the data and used to forecast the S&P Index for the next time period. This model fitting will take about three weeks.

Construction and training of the neural network, the third phase, will take another three weeks. The neural network will be trained on the same subsets of data and also used to make a forecast. Learning in a neural network involves two phases: the forward phase and the backward phase. During the forward phase, the input is propagated forward through the network to obtain an output value for each neuron and the difference between the desired output and the current output is computed. In the backward phase, the computed error is used and weights are changed in proportion to the error times the input signal. This is run backwards in order to tell how strongly a particular neuron is connected. The data is then sent through the network, and the process continues until the error is below some specified level. For a detailed explanation of the training process, see Malliaris and Salchenberger (1992).

As a last phase, these sets of forecasts will be compared against the actual S&P 500 Index prices using three criteria: the direction of the prediction, the mean square error and the mean absolute deviation. To compare and analyze the results from the models and obtain the results should take two to three weeks. If indeed the random walk holds and no hidden deterministic structure exists which can be captured by the nine explanatory variables, the neural network approach would not be able to outperform the random walk as judged by these three criteria. The results will then be prepared for publication and an article is expected by December, 1993.

The implications of this investigation are quite significant for several reasons. First, the neural network results will give evidence as to the appropriateness of one of the two alternative paradigms, random walk or chaotic. As White (1989) has suggested, neural networks could advance our empirical understanding of applied disciplines. Second, support for the chaotic paradigm would imply that active management of an S&P 500 portfolio is possible since the S&P follows a non-linear deterministic model. And third, if a neural network can outperform the random walk, then researchers would be encouraged to search for expressions linking the unknown but deterministic pattern of the S&P 500 to the explanatory variables.

3. APPENDIX -- Technical discussion of the Random Walk model and the Chaotic Dynamics Methodology

THE RANDOM WALK MODEL

Random walk is a statistical term used to describe the dynamic behavior. In its simplest formulation we define the sequence of prices, denoted by {p(t): t = 0, 1, 2, ...}, to follow a random walk if

$$p(t+1)=p(t)+\epsilon(t+1) \tag{1}$$

where ϵ(t+1) is the value obtained from sampling with replacement from a certain distribution with a given population mean μ and a variance of σ^2. Equation (1) expresses tomorrow's price as a random departure from today's price, or equivalently, the price change between today and tomorrow, i.e. p(t+1) - p(t), as random. It is usually assumed that $\mu = 0$.

The notion of a random walk has its methodological foundation in probability theory. Probability theory analyzes events whose outcome is uncertain in contrast to deterministic calculus where a relationship between the dependent and independent variables is exact. Long before the efficient market hypotheses was conceived, formulated and tested, the random walk model was utilized to convey the notion that stock prices cannot be systematically forecasted (Cowles(1933), Roberts(1959)). Roberts constructs a "chance model" and argues that weekly changes of a typical stock market index behave as if they were independent sample observations from a normal distribution with mean +0.5 and standard deviation 5.0.

The early observations of the random behavior of stock market prices and their modeling using the random walk paradigm, eventually directed researchers to seek explanations for such a statistical phenomenon. Thus, the efficient market hypotheses was developed to rationalize the random walk behavior claiming that the current price p(t) fully and correctly reflects all relevant information and because the flow of information between now and next period cannot be anticipated, price changes are serially uncorrelated.

Though numerous studies were done confirming market efficiency, several studies rejected it. The rejections of the random walk paradigm were considered to be of limited importance by most researchers and were called "anomalies". However, the October 1987 stock market crash caused a serious reevaluation of the efficient market hypotheses. Actually, numerous authors are currently expressing their skepticism about the ability of the theory to explain such a major decline in the absence of any fundamental change. Shleifer and Summers (1990) are quite critical and they claim that "the stock in the efficient market hypotheses -- at least as it has been traditionally formulated -- crashed along with the rest of the market on October 19, 1987".

THE CHAOTIC DYNAMICS METHODOLOGY

If the random walk model is not a satisfactory description of stock price behavior and if prices move without any obvious change in the fundamentals of the economy, what methodological alternatives exist to explain the observed price patterns? To answer this question, a handful of quantitative economists investigated the deterministic methods of Ruelle and Takens (1971) who studied the physical problem of turbulence. These authors and the numerous physicists who followed them developed a very active field of current research called chaotic dynamics. Chaotic dynamics yields a time series behavior that appears random when in fact such a series is generated by a nonlinear deterministic equation of high degree. Sometimes small input changes can produce very divergent outputs, causing a seemingly orderly system to become chaotic. The chaos appears to be random and unpredictable while it is actually following strict mathematical principles. Although deterministic, chaos is statistically equivalent to white noise.

Consider a real-valued function $f:R \rightarrow R$. We are interested in the time series generated by this function starting from some arbitrary $x_0 \in R$. Denote by $f^2 \equiv f[f(x)] \equiv f \circ f(x)$ where o means composition and in general let $f^n = f \circ f \circ \ldots \circ f(x)$ mean n compositions. The time series takes the values

$$x_o,\ f(x_o),\ f^2(x_o),\ \ldots,\ f^n(x_o),\ \ldots, \qquad (2)$$

for $t = 0, 1, 2, \ldots, n$. For (2) to describe a chaotic function it must satisfy three requirements.

First it must sample infinitely many values. The second requirement is sensitive dependence on initial conditions. This condition says that there are time series that start very close to each other but diverge exponentially fast from each other. The third requirement involves a property of the periodic points of the function f, namely that these periodic points are dense in R.

The methodology for detecting chaotic dynamics in stock price changes frequently uses the Grassberger and Procaccia (1983) correlation integral to compute the correlation dimension. Several studies have computed the correlation dimension for the S&P 500 Index. For example, Scheinkman and LeBaron (1989) concluded that the correlation dimension for the S&P 500 Index appeared to be about 6, implying that such an index has nonlinear structure. The investigation of chaotic systems can become extremely complicated. But the evidence of some unknown underlying structure motivates the search for nonlinear behavior through neural networks [White (1988)].

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