Nonlinear Techniques in Volatility Forecasting: A Neural Network Example

Dr. Mary Malliaris
Department of Management Science
Loyola University Chicago

Conference on Quantitative Methods in Financial Modeling

Varese, Italy
June 21, 1994

IS THE BEHAVIOR OF THE MARKET STRUCTURED?

2 answers

yes

no

if yes, must state structure and supply an equation

Since no one knows that, the currently accepted answer is no

THE RANDOM WALK MODEL

RANDOM WALK IS A STATISTICAL TERM USED TO DESCRIBE DYNAMIC BEHAVIOR

shock

$$p(t+1) = p(t) + \epsilon(t+1)$$

a numerical expression of information

where E(shock) = 0

THE EFFICIENT MARKET HYPOTHESIS STATES THAT

THE CURRENT PRICE $p(t+1)$ FULLY AND CORRECTLY REFLECTS ALL RELEVANT INFORMATION

AND

THE CURRENT PRICE IS THE BEST PREDICTOR OF TOMORROW'S PRICE

Those who believe the market is structured

use THE CHAOTIC DYNAMICS METHODOLOGY *

⇒CHAOTIC DYNAMICS PROPOSES A TIME SERIES BEHAVIOR THAT APPEARS RANDOM WHEN IN FACT SUCH A SERIES IS GENERATED BY A NONLINEAR DETERMINISTIC EQUATION ■

SOME TESTS HAVE IMPLIED THAT THE S&P 500 MAY NOT BE RANDOM, BUT THEY ARE NOT CONCLUSIVE. FOR EXAMPLE, SCHEINKMAN AND LEBARON (1989) CONCLUDED IN ONE TEST THAT THE CORRELATION DIMENSION FOR THE S&P 500 INDEX APPEARS TO BE ABOUT 6, IMPLYING THAT THE INDEX HAS NONLINEAR STRUCTURE.

* They are inspired by the work of Lorenz who produced a system of deterministic differential equations whose paths look like a random walk. ⇒

■ So, given real world data (that looks random) how can we distinguish whether the underlying mechanism is random or deterministic?

There are some tests which imply the S+P 500 may have a structure, but they are not conclusive (i.e. necessary but not sufficient)

IMPLICATIONS OF THIS INVESTIGATION:

1. THE NEURAL NETWORK RESULTS WILL GIVE EVIDENCE AS TO THE APPROPRIATENESS OF ONE OF THE TWO ALTERNATIVE PARADIGMS, RANDOM WALK OR CHAOTIC

2. SUPPORT FOR THE CHAOTIC PARADIGM WOULD IMPLY THAT ACTIVE MANAGEMENT OF AN S&P PORTFOLIO IS POSSIBLE

3. IF A NEURAL NETWORK CAN OUTPERFORM THE RANDOM WALK, RESEARCHERS WOULD BE ENCOURAGED TO SEARCH FOR EXPRESSIONS LINKING THE UNKNOWN BUT DETERMINISTIC PATTERN OF THE S&P 500 TO THE EXPLANATORY VARIABLES

DATA

WEEKLY DATA FROM EACH FRIDAY FOR TWO YEARS (89-90)

TEN VARIABLES:

1. THE S&P 500 CLOSING INDEX
2. THE THREE MONTH TREASURY BILL INTEREST RATE
3. THE THIRTY YEAR TREASURY BOND INTEREST RATE
4. WEEKLY NEW YORK STOCK EXCHANGE VOLUME
5. MONEY SUPPLY AS MEASURED BY M1
6. MONEY SUPPLY AS MEASURED BY M2
7. PRICE/EARNINGS RATIO
8. THE PRICE OF GOLD
9. THE PRICE OF CRUDE OIL
10. THE CBOE PUT/CALL RATIO

NETWORK INPUT VARIABLES:

EACH OF THE TEN DATA VALUES

TWO LAGS ON EACH OF THE TEN DATA VALUES

WEEK OF THE MONTH

MONTH OF THE YEAR

NETWORK OUTPUT VARIABLE:

THE NEXT FRIDAY'S VALUE OF THE S&P CLOSING INDEX

BACKPROPAGATION NETWORK DEVELOPED USING

BrainMaker Professional ®

BY CALIFORNIA SCIENTIFIC SOFTWARE

and BrainMaker's Genetic Training Option

DETERMINING THE HIDDEN LAYERS
USING BrainMaker's GENETIC TRAINING OPTION (GTO)

GTO ENABLES YOU TO DETERMININE BOTH

* THE OPTIMUM NUMBER OF HIDDEN NEURONS

AND

* THE OPTIMUM NUMBER OF HIDDEN LAYERS

	FROM	TO	STEP
HIDDEN NEURONS, LAYER 1	______	______	_____
HIDDEN NEURONS, LAYER 2	______	______	_____
HIDDEN LAYERS:	ONE AND TWO		

FOR EACH CONFIGURATION, BrainMaker GENERATES A NETWORK AND TRAINS IT TO A FIXED NUMBER OF TRAINING RUNS, TESTING IT EVERY N RUNS

GTO ACCUMULATES A STATISTICS FILE ON EVERY NETWORK WITH

* RMS ERROR
* MEAN ABSOLUTE ERROR
* R^2
* THE NUMBER OF GOOD FACTS
* THE NUMBER OF THE RUN

THIS FILE CAN BE SORTED ON ANY OF THESE STATISTICS

YOU CAN NOW SELECT THE BEST CONFIGURATION

USING GTO TO FIND THE BEST INITIAL TRAINING WEIGHT MATRIX

GTO'S GENETIC EVOLUTION USES

*MUTATION
*CROSSOVER

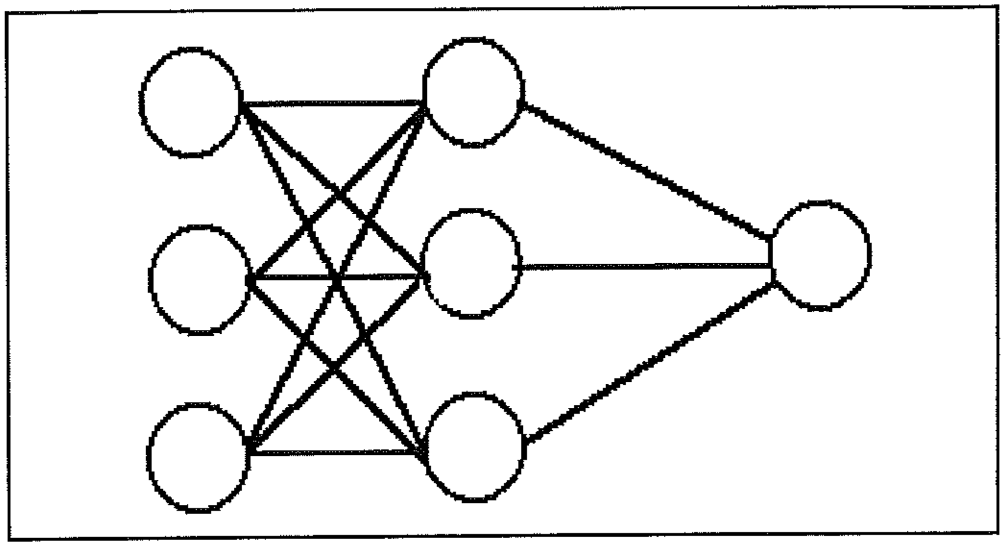
TO OPTIMIZE THE MATRIX OF INITIAL WEIGHTS WHICH WILL BE USED IN THE TRAINING OF THE NETWORK

NEURAL NETWORK MUTATION

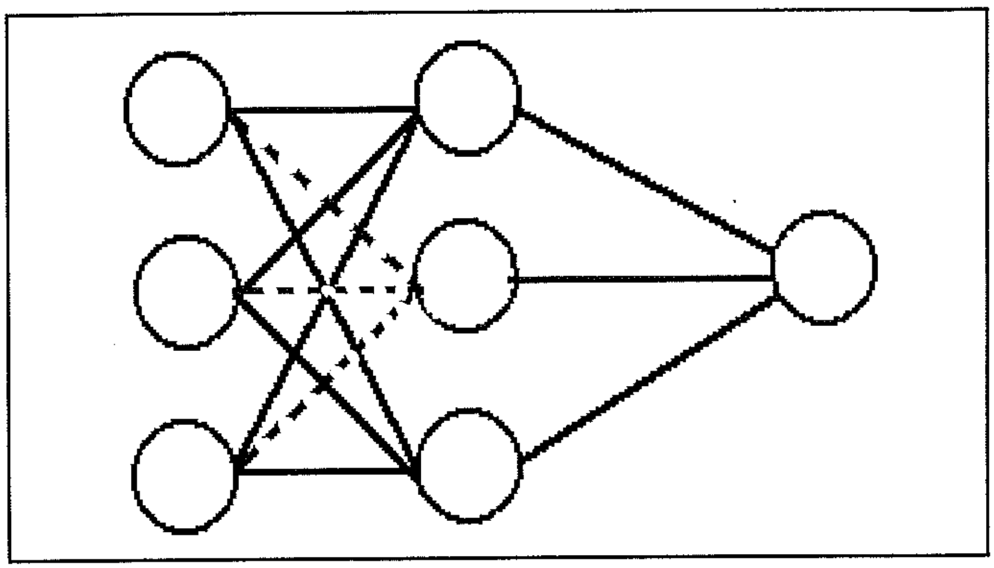
A RANDOM PERCENTAGE OF THE NEURONS IS CHANGED BY MODIFYING THE WEIGHTS ASSOCIATED WITH THEM

THE FREQUENCY OF CHANGE AND THE AMOUNT WHICH THE WEIGHTS ARE CHANGED CAN BE SET BY THE USER

PARENT



CHILD

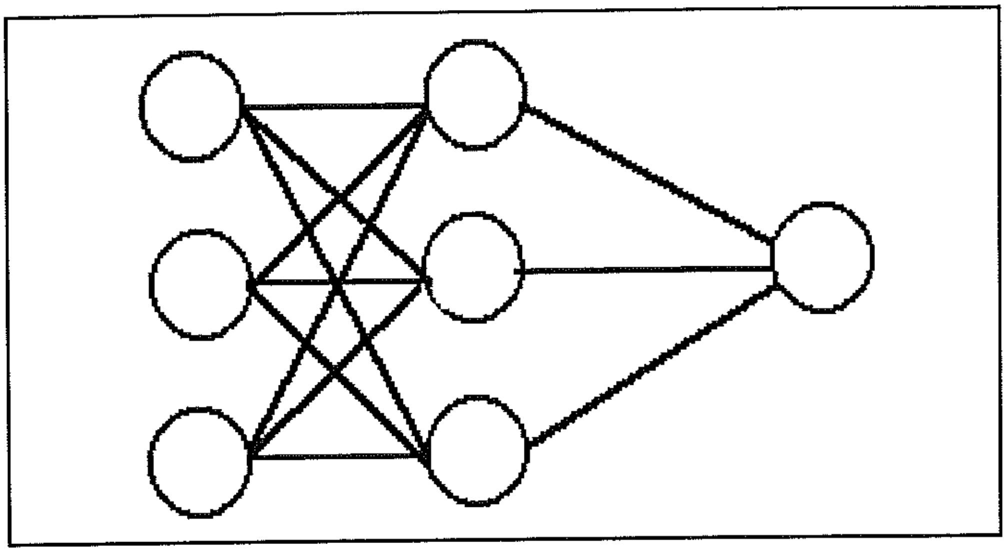


Mutation

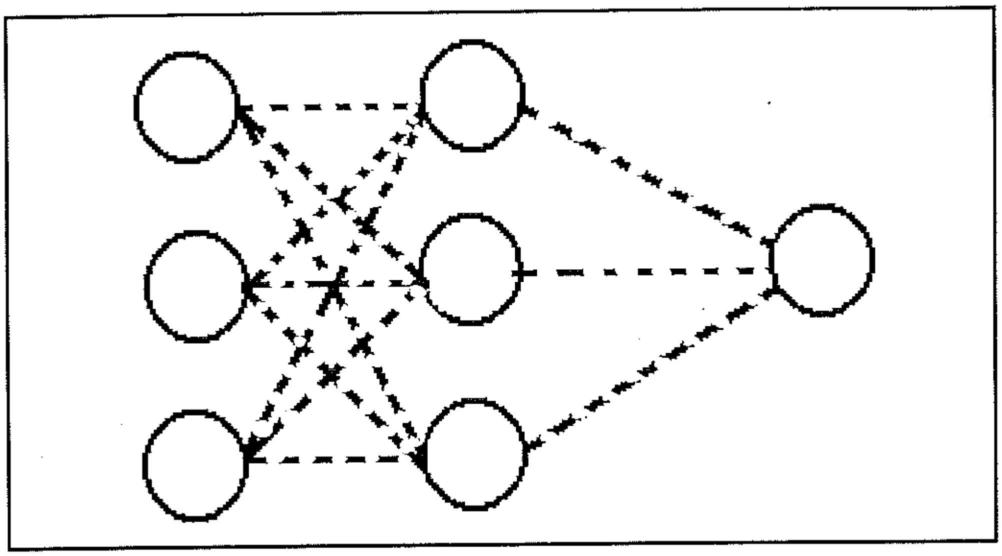
NEURAL NETWORK CROSSOVER

(REQUIRES 2 PARENTS) SOME NEURONS ARE TAKEN FROM EACH PARENT AND USED TO PRODUCE A CHILD

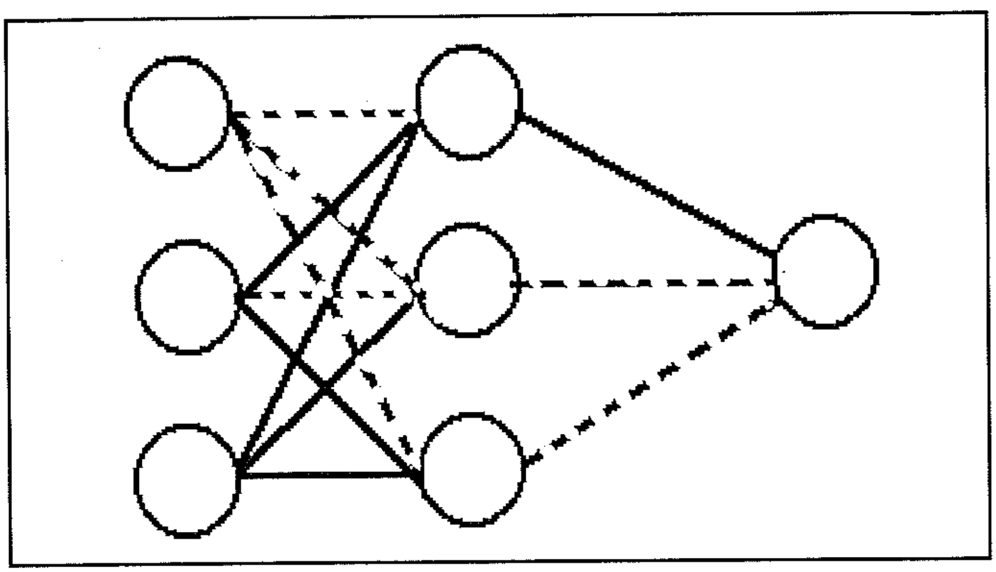
PARENT 1



PARENT 2



CHILD



CROSSOVER

GTO THEN "POLISHES" THE CHILD BY TRAINING IT FOR A CERTAIN NUMBER OF RUNS

NEXT, THE CHILD NETWORK IS TESTED AND THE RESULTS ARE COMPARED TO THE PARENT

IF IT DOES BETTER, THE CHILD REPLACES THE PARENT WHICH IT HAS OUTPERFORMED

CHILDREN ARE PRODUCED TO THE NUMBER OF USER-SPECIFIED GENERATIONS

THE BEST SET OF CHILD WEIGHTS IS SAVED AS GTO001

THIS MATRIX OF WEIGHTS IS THEN USED FOR THE STARTING POINT OF TRAINING AND TESTING THE DATA IN BrainMaker

FOR THE S&P 500

	FROM	TO	STEP
HIDDEN NEURONS, LAYER 1	2	45	1
HIDDEN NEURONS, LAYER 2	2	45	1
HIDDEN LAYERS:	ONE AND TWO		

BEST CONFIGURATION ON BOTH RMSerror AND R^2

* 2 HIDDEN LAYERS

32 inputs
1 output

* 27 NODES IN HIDDEN LAYER ONE

* 8 NODES IN HIDDEN LAYER TWO

THE WEIGHTS WERE BOTH MUTATED AND CROSSED OVER FOR 20 GENERATIONS

AFTER EVOLVING THE INITIAL WEIGHTS, THE TEN DATA SETS CONVERGED AT AN AVERAGE OF 24 RUNS

Save this for later maybe

32 input var

Before GTO : 250 runs to converge

Hidden Layers : 150

Weight : 24

CROSS-VALIDATION

FOR ASSESSING THE TRUE UNBIASED AMOUNT OF ERROR IN A NEURAL NETWORK MODEL

THE DATA SET IS DIVIDED INTO K DISTINCT SETS OF ABOUT THE SAME SIZE

EACH SET IS USED INDEPENDENTLY FOR TESTING WHILE THE REMAINING DATA IS UTILIZED FOR TRAINING THE NETWORK

EACH TESTING SET WILL HAVE A FINAL ERROR AMOUNT

THE AVERAGE OF THESE ERRORS OVER ALL THE K SETS IS THE ESTIMATE OF THE TRUE ERROR RATE

THE S&P 500

TWO YEARS OF WEEKLY DATA, WITH LAGS

DIVIDED INTO TEN SETS, A THRU J

EACH SET WAS WITHHELD AS A TESTING SET AND A NETWORK WAS TRAINED ON THE REMAINING DATA

THE EFFICIENT MARKET HYPOTHESIS:

THE BEST ESTIMATE OF A VALUE FOR THE FOLLOWING PERIOD IS EXPECTED TO BE THE VALUE IN THE CURRENT PERIOD

USING THE SAME TEN TESTING PERIODS AS IN THE NETWORK CROSS-VALIDATION,

* MEAN ABSOLUTE DEVIATION
* MEAN SQUARE ERROR
* CORRELATION BETWEEN EXPECTED AND ACTUAL OUTPUT

WERE CALCULATED

COMPARISON STATISTICS FOR OUTPUT FROM NEURAL NETWORKS AND RANDOM WALK

Data Set	MAD	MSE	Correlation
A network	2.633	14.083	0.9895
random walk	6.155	53.203	0.9629
B network	2.123	5.897	0.9965
random walk	4.887	30.567	0.9580
C network	2.506	10.236	0.9848
random walk	6.552	61.250	0.9020
D network	2.905	10.449	0.9894
random walk	3.716	21.952	0.9876
E network	4.072	28.987	0.9710
random walk	5.653	46.875	0.9644
F network	3.087	13.132	0.9945
random walk	3.496	22.869	0.9836
G network	4.000	21.396	0.9595
random walk	6.539	50.489	0.9385
H network	3.679	21.184	0.9723
random walk	4.806	29.089	0.9708
I network	3.548	17.959	0.9750
random walk	6.111	61.577	0.9202
J network	3.060	12.718	0.9810
random walk	5.036	35.194	0.9465
averages N	3.167	15.609	
R	6.188	41.32	

CONCLUSIONS

USING TWO YEARS WORTH OF WEEKLY DATA, A BACKPROPAGATION NEURAL NETWORK WAS DEVELOPED WHICH PERFORMED BETTER THAN THE RANDOM WALK HYPOTHESIS

THIS SUPPORTS THOSE WHO BELIEVE A DETERMINISTIC STRUCTURE EXISTS IN THE S&P 500

THE RESULTS ARE ENCOURAGING TO RESEARCHERS WHO WISH TO DEVELOP DETERMINISTIC THEORIES WHICH MAY EVENTUALLY REPLACE THE EXISTING PROBABILISTIC PARADIGM