VOLUME AND PRICE RELATIONSHIPS: HYPOTHESES AND TESTING FOR AGRICULTURAL FUTURES

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1. INTRODUCTION

The relationship between trading volume and price variability has been examined extensively. The theoretical motivation of earlier studies such as Ying (1966), Crouch (1970), Clark (1973), Copeland (1976), Epps and Epps (1976), Westerfield (1977), Rogalski (1978), and Upton and Shannon (1979) was the demand and supply model of microeconomic theory. Some authors have investigated the price–volume relationship with the use of data from futures markets; these include Cornell (1981), Tauchen and Pitts (1983), Rutledge (1984), Grammatikos and Saunders (1986), Garcia, Leuthold, and Zapata (1986), and Bhar and Malliaris (1996). Other researchers have studied the determinants of volume with the use of macroeconomic and financial variables other than price vari-

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ability. Key references in this direction of research are Carlton (1983, 1984) and Martell and Wolf (1987). Theoretical models of trading volume have been developed also by Karpoff (1986), Huffman (1987), and Pagano (1989).

Researchers have emphasized the importance of the relationship between price and volume. Karpoff (1987) gives several reasons why the price–volume relationship is crucial in capital markets. He argues that the price–volume relationship can provide insight about the market structure, because information is more available for heavily traded securities than for thinly traded securities. Also, larger volumes make trade more competitive and lower the bid–ask spread. Trading volume also plays an important role in futures markets. Most economic reports published by the futures exchanges and regulatory agencies use volume data to measure the growth or decline of futures contracts. Volume data are also used to measure shifts in the composition of futures markets.

Furthermore, volume is of great significance in technical analysis. Unlike the efficient market hypothesis, which underscores the importance of asset prices and claims that prices fully incorporate all relevant information, technical analysis extends this notion to volume as well. Murphy (1985) and DeMark (1994) emphasize that both volume and price incorporate valuable information. Bullish news causes not only prices to increase, but also trading volume. A technical analyst gives less significance to a price increase with low trading volume than to a similar price increase with substantial volume.

Finally, some authors, such as Peck (1981), study the role of speculation and price volatility. Speculation is closely related to trading volume. Although the study of price volatility can be carried out without reference to volume, as in Streeter and Tomek (1992), most often these two variables are linked together, as in Cornell (1981).

This article contributes to the literature of price–volume relationship and the determinants of trading volume by postulating several hypotheses and testing them with data for agricultural commodity futures contracts. The model developed in this article formalizes the intuitive idea that price and quantity are interrelated. The theoretical model presented in the article differs from the earlier works of Crouch (1970), Rogalski (1978), Martell and Wolf (1987), Karpoff (1986), Huffman (1987), and Pagano (1989) by using stochastic calculus and Itô's processes. The empirical portion of the article differs from previous research in several aspects. Clark (1973), Rutledge (1978), Cornell (1981), Tauchen and Pitts (1983), and Grammatikos and Saunders (1986) concentrate on the investigation of the relationship between volume and price volatility. Martell and Wolf (1987) examine the determinants of trading volume. Garcia et al. (1986) investigate lead–lag relationships between trading volume and price variability. This article conducts tests of long-run relationships, or tests of cointegration, between price and volume, and also applies an error correction model to volume and price. Finally, tests of the determinants of trading volume are also reported. All these tests use an extended data set that covers the time period 1981–1995.

The remainder of the article is organized as follows: Section 2 presents the postulated model and hypotheses; Section 3 describes the methodology; Section 4 presents the data; Section 5 analyzes the empirical results, and Section 6 summarizes and concludes the article.

2. MODEL AND HYPOTHESES

Following the previous work of Crouch (1970), Rogalski (1978), Garcia et al. (1986), and Bhar and Malliaris (1996), it is postulated that volume is a function of price and time:

$$V = V(t, P) \tag{1}$$

where V denotes trading volume, P denotes futures price, and t denotes time. The relationship between volume and price can indeed be highly complicated, and it can dynamically change over time. This change over time is expressed by the argument, t, in (1). In other words, expression (1) goes beyond the static supply and demand model by emphasizing a dynamic relationship. This is more appropriate for futures markets where the price–quantity relationship changes almost continuously.

Assume that the function, V, in (1) is twice continuously differentiable and that P follows an Itô process with drift, μ, and volatility, σ, written as

$$dP = \mu\, dt + \sigma\, dZ \tag{2}$$

In (2), Z denotes a standardized Weiner process. The appropriateness of (2) to describe asset prices is reviewed extensively in Merton (1982), who offers arguments in support of the use of Itô processes to characterize the behavior of asset prices. Among these arguments, the most compelling one is that Itô processes describe continuous random walks with a drift.

An application of Itô's lemma presented in Malliaris and Brock (1982) yields

$$dV = V_t + V_P\,dP + \frac{1}{2}V_{PP}\,(dP)^2$$

$$= V_t\,dt + V_P\,[\mu\,dt + \sigma\,dZ] + \frac{1}{2}V_{PP}\,\sigma^2\,dt$$

$$= [V_t + V_P\,\mu + \frac{1}{2}V_{PP}\,\sigma^2]\,dt + V_P\,\sigma\,dZ \tag{3}$$

where V_t, V_P, and V_{PP} denote partial derivatives. The relationships described by eqs. (1)–(3) allow one to formulate several hypotheses.

Observe that both P and V in (1)–(3) are random variables with certain distribution functions. If these distribution functions change over time, then V and P are nonstationary. Also, eq. (2) describes futures prices as a diffusion process. Because diffusion processes are continuous-time random walks, eqs. (1)–(3) claim the following: If futures prices follow a random walk, then trading volume also follows a random walk. Tests of randomness and stationarity for both price and volume allow verification of the validity of this first hypothesis.

Secondly, eqs. (1)–(3) suggest that futures price and the corresponding trading volume are interrelated and can affect each other. Cointegration and error correction methodologies are used to test this second hypothesis—that price and volume relate to each other in the long run and in the short run.

If the expectations of (3) are taken into account, the following expression is derived:

$$E\,(dV) = V_t + V_P\,\mu + \frac{1}{2}V_{PP}\,\sigma^2 \tag{4}$$

Equation (4) suggests that the change in trading volume depends on three determinants: (i) a trend factor, V_t; (ii) the drift coefficient of price, μ; and (iii) the volatility of price, σ^2. This third hypothesis is tested with the following expression:

$$E\,(dV) = \alpha\,t + \beta\,\mu + \gamma\,\sigma^2 \tag{5}$$

Finally, stochastic calculus techniques allow derivation of the volatility of trading volume from (3) as

$$Var\,(dV) = V_P^2\,\sigma^2, \tag{6}$$

which says that the volatility of trading volume is a function of price volatility. This is the fourth hypothesis of this article and is tested with the following relationship:

$$Var\,(dV) = \alpha + \delta\,\sigma^2 \quad (7)$$

3. METHODOLOGY

The four hypotheses are tested with augmented Dickey and Fuller tests of stationarity, tests of cointegration, and the error correction methodology. Brief descriptions of these methods follow.

3.1 Tests of Stationarity

The stationarity of price and trading volume is tested with the augmented Dickey and Fuller (ADF) (1979), test:

$$X_t - X_{t-1} = b_0 X_{t-1} + \sum_{i=1}^{T} b_i (X_{t-i} - X_{t-i-1}) + \varepsilon_t \quad (8)$$

where X_t represents the level or the first difference of the variables. The null hypothesis of nonstationarity is $b_0 = 0$. If the null hypothesis cannot be rejected for the level of the variable but is rejected for the first difference, then the variable is stationary in the first difference and it is said that the variable is integrated of order 1, denoted by I(1).

3.2 Tests of Cointegration

If two time series, X_t and Y_t, are both nonstationary in levels but stationary in the first difference, it is said that variables, X_t and Y_t, are integrated of order 1, denoted as I(1). If two variables, X_t and Y_t, are both I(1), their linear combinations, $Z_t = X_t - \alpha Y_t$, are generally also I(1). However, if there is an α such as that Z_t is I(0), then Z_t is integrated of order 0 or stationary in level. If Z_t is I(0), then the linear combination of X_t and Y_t is stationary and it is said that the two variables are cointegrated. Cointegration represents a long-run equilibrium relationship between two variables.

Engle and Granger (1987) propose several methods to test for cointegration between two time series. This study follows the approach of first running the cointegration regression:

$$X_t = \alpha_0 Y_t + \varepsilon_t \tag{9}$$

and then running the ADF regression

$$\varepsilon_t - \varepsilon_{t-1} = b_0 \varepsilon_{t-1} + \sum_{i=1}^{T} b_i(\varepsilon_{t-i} - \varepsilon_{t-i-1}) + \mu_t \tag{10}$$

on the residuals of (9). The null hypothesis of no cointegration is H_0: $b_0 = 0$. If the null hypothesis is rejected, then the variables, X_t and Y_t, are cointegrated and there is some long-term relationship between them.

In addition to the cointegration methodology described in (9) and (10) the maximum likelihood method is used to estimate the cointegrating relationship between variables developed by Johansen (1988). The Johansen methodology assumes that X_t in (11) is an unrestricted vector autoregressive (VAR) process of N variables:

$$X_t = \Pi_1 X_{t-1} + \Lambda + \Pi_k X_{t-k} + \varepsilon_t \tag{11}$$

where each II is an $N \times N$ matrix of parameters. The system in (11) can be expressed in the error correction form (ECM) as

$$\Delta X_t = \Gamma_1 \, \Delta X_{t-1} + \Gamma_2 \, \Delta X_{t-2} + \Lambda + \Gamma_{k-1} \, X_{t-k+1} + \Gamma_k \, \Delta X_{t-k} + \varepsilon_t \tag{12}$$

where

$$\Gamma_i = -I + \Pi_1 + \Pi_2 + \Lambda - \Pi_i, \qquad i = 1,2,\Lambda k$$

If X_t is a vector of I(1) variables, then the left-hand side and the first $(k - 1)$ elements on the right-hand side of (12) are I(0) and the kth term is a linear combination of I(1) variables. Johansen shows by use of a canonical correlations method how to estimate all the distinct combinations of levels of X that produce high correlations with the stationary elements of (12). These combinations are the cointegrating vectors. Johansen (1991) also shows how to test which of these distinct cointegrating vectors are statistically significant and derives critical values for this test.

3.3 Granger Causality and Error Correction Model (ECM)[1]

A time series, Y_t, causes another time series, X_t, if the current value of X can be predicted better by using past values of Y than by not doing so,

[1]The authors are grateful to Professor Hector Zapata for his guidance and instruction in this section.

considering also other relevant information, including past values of X. Specifically, Y is causing X if some coefficient, a_i, is not zero in the following equation:

$$X_t - X_{t-1} = c_0 + \sum_{i=1}^{T} a_i(Y_{t-i} - Y_{t-i-1}) + \sum_{j=1}^{T} b_j(X_{t-j} - X_{t-j-1}) + \varepsilon_t \qquad (13)$$

Similarly, X is causing Y if some coefficient, α_i, is not zero in eq. (14):

$$Y_t - Y_{t-1} = \gamma_0 + \sum_{i=1}^{T} \alpha_i(X_{t-i} - X_{t-i-1}) + \sum_{j=1}^{T} \beta_j(Y_{t-j} - Y_{t-j-1}) + \mu_t \qquad (14)$$

If both events occur, there is a feedback. T is the number of lags for the variable, selected with the use of the Akaike criterion.[2]

By integrating the concepts of cointegration and causality in the Granger sense, it is possible to develop a model that allows for the testing of the presence of both a short-term and a long-term relationship between the variables, X_t and Y_t. This model is known as the error correcting model (ECM) proposed by Engle and Granger (1987) and discussed in numerous papers. Key recent references include Zapata and Rambaldi (in press) and Giannini and Mosconi (1992). In particular, Zapata and Rambaldi (in press) provide Monte Carlo evidence for tests based on maximum-likelihood estimation of ECM. They confirm that in large samples all tests perform well in terms of size and power. Because sample size of this study has 3,649 observations, there are no small sample problems.

In (15), the ECM model investigates the potential long-run and short-run impact of the variable, Y_t, on the variable, X_t:

$$X_t - X_{t-1} = a_1\hat{Z}_{t-1} + \sum_{i=1}^{T} c_i(Y_{t-i} - Y_{t-i-1}) + \sum_{j=1}^{T} d_j(X_{t-j} + X_{t-j-1}) + \varepsilon_t \qquad (15)$$

[2]The Akaike criterion suggested the use of three lags for the variables used.

The ECM model represented by eq. (15) decomposes the dynamic adjustments of the dependent variable, X_t, to changes in the independent variable, Y_t, into two components: first, a long-run component given by the cointegration term, $a_1 \hat{Z}_{t-1}$, also known as the error correction term, and second, a short-term component given by the first summation term on the right-hand side of eq. (15). Observe the difference between eq. (13) and (15), namely, the cointegration term, $a_1 \hat{Z}_{t-1}$, is added in eq. (15). Recall from the discussion preceding (9) that $\hat{Z}_t = X_t - \alpha_0 Y_t$.

Similarly, the long-run and short-run impact of X_t on Y_t can be captured by the following ECM model:

$$Y_t - Y_{t-1} = \beta_1 \hat{Z}_{t-1} + \sum_{i=1}^{T} \phi_i (X_{t-i} - X_{t-i-1}) + \sum_{j=1}^{T} \theta_j (Y_{i-j} + Y_{t-j-1}) + \varepsilon_t \qquad (16)$$

From eqs. (15) and (16) one may deduce that the variables, X_t and Y_t, exhibit long-run movements when at least one of the coefficients, a_t or β_1, is different from zero. If a_1 is statistically different from zero but β_1 is not, then the implication is that X_t follows and adjusts to Y_t in the long run. The opposite occurs when β_1 is statistically different from zero but a_1 is not. If both coefficients, a_1 and β_1, are statistically different from zero, a feedback relationship exists, implying that variables, X_t and Y_t, adjust to one another over the long run.

The coefficients, c_i's and ϕ_i's, in eqs. (15) and (16), respectively, represent the short-term relationships between the variables, X_t and Y_t. If the c_i's are not all zero in a statistical sense but all ϕ_i's are, then Y_t is leading or causing X_t in the short run. The reverse case occurs when the ϕ_i's are not all zero in a statistical sense but all c_i's are. If both events occur, then there is a feedback relationship and the variables, X_t and Y_t, affect each other in the short run.

3.4 Tests of the Determinants of Trading Volume

Expressions (4) and (5) are implemented by running the following regressions:

$$\Delta V_t = \alpha_0 + \alpha_1 t + \beta(\Delta P_t) + \gamma |\Delta P_t| \qquad (17)$$

where

$\Delta V_t = V_t - V_{t-1}$, change in trading volume

t = time trend

$\Delta P_t = P_t - P_{t-1}$, change in price

$|\Delta P_t|$ = absolute change in price as a measure of price volatility

The following regression is used to empirically test eqs. (6) and (7):

$$|\Delta V_t| = \alpha + \delta\,|\Delta P_t| \tag{18}$$

where

$|\Delta V_t|$ = absolute change in volume as a measure of trading volume's volatility.

$|\Delta P_t|$ = absolute change in price as a measure of price volatility

4. DATA

The data correspond to daily settlement prices and trading volume for six agricultural futures contracts: corn, wheat, oats, soybean, soybean meal, and soybean oil, provided by Knight-Ridder Financial. The data sample covers the time period from January 2, 1981 through September, 29, 1995. There are a total of 3649 observations for prices and volumes for each of the six agricultural futures. The prices are for the nearby contract, and the trading volume corresponds to the nearby plus the more distant contracts. At the expiration of a given futures contracts, the price reported refers to the new nearby contract.[3]

5. ANALYSIS OF EMPIRICAL RESULTS

The first empirical issue investigated in this article is the time-series properties of price and volume of trade. Tables I and II present the augmented Dickey and Fuller tests of stationarity. The number of lags used in the test of stationarity cointegration, and error correction are determined by using the Akaike information criterion.[4] The null of nonstationarity cannot be rejected for the levels of price and trading volume, but it is strongly

[3]A referee raised the question of possibly abnormal returns between the price of the expiring contract and the price of the new nearest by contract. Such returns could bias the results due to jumps. To address this issue, all tests are run twice: once with data containing a possible jump at expiration and once by smoothing such jumps with the use of the last three observations from the expiring contract and the first three observations from the new contract and averaging these six to reduce the jump. The difference in the results between these two sets of data are fortunately insignificant.

[4]Recall remarks in Footnote 2.

TABLE I

Augmented Dickey-Fuller Tests of Stationarity for Prices

	Price Level		*Price First Difference*	
Commodity	b_0, *t stat*	R^2, *F stat*	b_1, *t stat*	R^2, *F stat*
Corn	−0.0002 (−1.0308)	0.021474 (26.63458)	−0.890166 (−30.12368)	0.428502 (909.7432)
Wheat	−0.000198 (−0.968202)	0.011331 13.90953	−0.892157 (−28.92019)	0.452767 (1003.882)
Oats	−0.000331 (−1.101373)	0.026696 (33.28851)	−0.813420 (−28.38429)	0.421430 (883.7920)
Soybean	−0.000210 (−0.887785)	0.002314 (2.814699)	−0.996215 (−30.53370)	0.484499 1140.365
Soybean meals	−0.000216 (−0.872463)	0.004851 (5.916781)	−0.973412 (−30.34343)	0.475030 (1097.909)
Soybean oil	−0.000126 (−0.478395)	0.012044 14.79532	−0.918213 (−29.37228)	0.451168 (997.4227)

Notes: The model is

$$\Delta X_t = b_0 X_{t-1} + \sum_{i=1}^{T} b\,\Delta_t X_{t-i} + \varepsilon_t.$$

The null hypothesis is $\mathbf{H}_0$: $b_0 = 0$ (X_t is not stationary). The MacKinnon critical values for rejection of the null hypothesis are 1% critical value = −2.57, 5% critical value = −1.94, 10% critical value = −1.62. The *t* statistics and the *F* statistics are given in parentheses.

TABLE II

Augmented Dickey-Fuller Tests of Stationarity for Volume

	Volume Level		*Volume First Difference*	
Commodity	b_0, *t stat*	R^2, *F stat*	b_1, *t stat*	R^2, *F stat*
Corn	−0.000132 (−0.323009)	0.216407 (335.1810)	−2.273773 (−45.00779)	0.714922 (3042.816)
Wheat	−0.000260 (−0.470215)	0.216504 (335.3734)	−2.428902 (−48.66764)	0.722048 (3151.925)
Oats	−0.000835 (−0.818090)	0.215610 (333.6086)	−2.249846 (−44.61281)	0.715478 (3051.124)
Soybean	−0.000175 (−0.432610)	0.260330 (427.1560)	−2.434077 (−46.33018)	0.741507 (3480.548)
Soybean meals	−0.000284 (−0.509007)	0.215100 (332.6033)	−2.351476 (−47.04102)	0.715977 (3058.621)
Soybean oil	−0.000251 (−0.459208)	0.212574 (327.6424)	−2.291039 (−45.82745)	0.715331 (3048.929)

Notes: The model is

$$\Delta X_t = b_0 X_{t-1} + \sum_{i=1}^{T} b\,\Delta_t X_{t-i} + \varepsilon_t.$$

The null hypothesis is $\mathbf{H}_0$: $b_0 = 0$ (X_t is not stationary). The MacKinnon critical values for rejection of the null hypothesis are 1% critical value = −2.57, 5% critical value = −1.94, 10% critical value = −1.62. The *t* statistics and the *F* statistics are given in parentheses.

TABLE III

Engle and Granger Test of Cointegration of Price and Volume

Commodity	*Dependent Variable (X)*	*Independent Variable (Y)*	b_0, *t stat*
Corn	Price	Volume	−0.022970 (−4.901632)
	Volume	Price	−0.207838 (−13.96623)
Wheat	Price	Volume	−0.032884 (−5.898205)
	Volume	Price	−0.273314 (−16.15283)
Oats	Price	Volume	−0.005926 (−3.268762)
	Volume	Price	−0.161456 (−12.22476)
Soybean	Price	Volume	−0.067062 (−7.769626)
	Volume	Price	−0.280663 (−15.89687)
Soybean meal	Price	Volume	−0.022256 (−4.737286)
	Volume	Price	−0.309020 (−17.22669)
Soybean oil	Price	Volume	−0.020450 (−4.495214)
	Volume	Price	−0.289937 (−16.74409)

Notes: The model is

$$X_t = a_0 + a_1 Y_{t-1} + \varepsilon_t$$

$$\Delta\varepsilon_t = b_0\varepsilon_{t-1} + \sum_{i=1}^{T} \Delta\varepsilon_{t-i} + \mu_t$$

The null hypothesis is $\mathbf{H}_0$: $b_0 = 0$ (X_t is not stationary). The MacKinnon critical values for rejection of the null hypothesis are 1% critical value = −2.57, 5% critical value = −1.94, 10% critical value = −1.62.

rejected for the first differences of the variables. It is concluded that price and volume of trade follow nonstationary random processes and are integrated of order one, I(1), which is a condition for testing for cointegration.

The tests of cointegration presented in Tables III and IV indicate the existence of long-term relationships between price and trading volume for the six agricultural commodity futures contracts. Observe from Table III that the relationship is stronger from price to volume, suggesting that trading volume tends to follow and adjust to price over the long run. These results are also confirmed with the use of the Johansen (1988, 1991)

TABLE IV

Johansen Test of Cointegration of Price and Volume

Likelihood-Ratio test

	Corn	Wheat	Oats	Soybeans	Soybean Meal	Soybean Oil
$r = 0$	13.82613*	29.49035**	25.13110**	17.91067**	19.74685**	13.17534*
$r \leq 1$	0.251170	0.432751	0.676079	0.284707	0.362984	0.103778

Cointegrating Vector Corresponding to the Largest Eigenvalue

	Corn	Wheat	Oats	Soybeans	Soybean Meal	Soybean Oil
Price	−0.000360	−0.000369	−0.000436	−0.000197	−0.000571	−0.003797
Volume	0.007502	0.011649	0.008203	0.010073	0.010861	0.008962

Notes: The 1% and 5% critical values for the Johansen test are 16.31 and 12.53 for $r = 0$ and 6.51 and 3.84 for $r \leq 1$, respectively, where r represents the number of cointegrating vectors. A value of the likelihood-ratio test statistic less than the corresponding 5% critical value implies that the corresponding hypothesis regarding r cannot be rejected. If $r = 0$ cannot be rejected the price and volume series are not cointegrated. If the price and volume series are cointegrated, then using the coefficients in the lower panel, a linear combination can be created which will be stationary.
*Null hypothesis rejected at the 5% confidence level.
**Null hypothesis rejected at the 1% confidence level.

methodology presented in Table IV. The likelihood-ratio test rejects the null hypothesis of no cointegration between price and volume for all six of the agricultural commodities. Thus, both cointegration methodologies offer strong evidence in support of the hypothesis that price and volume are interrelated.

Having established the existence of cointegration between price and volume for all six of the agricultural commodities, it is natural to test for causality. Observe that if there is cointegration between two variables, for sure there is causality in at least one direction. This implies information about instantaneous causality, in contrast to cointegration, which captures the long-run relationship. The idea is highlighted in Giannini and Mosconi (1992).[5]

The error-correction methodology allows the simultaneous study of the long-term and short-term impacts of one variable upon the other. Table V confirms that for all six contracts, a strong long-term relationship exists, both from price to volume and from volume to price. Strong long-term relationship means statistically significant in terms of both the t and F statistics. The t statistic identifies the significance of each coefficient

[5]An anonymous referee printed out the logical relationship between cointegration and causality.

TABLE V

Error-Correction Model (FCM) for Testing for Long-Term and Short-Term Relationship for Price and Volume of Agricultural Futures Contracts

	Dependent Variable	Independent Variable	a_1, β_1 (t stat)	c_1, ϕ_1 (t stat)	c_2, ϕ_2 (t stat)	c_3, ϕ_3 (t stat)	H_0: No Relationship* F stat Probability: No LT Impact	No ST Impact	No LT or ST Impact
Corn	Price	Volume	−0.003544 (−3.1947)	−0.355585 (−2.0996)	0.301002 (1.67314)	0.081684 (0.48425)	10.04098 (0.001544)	4.175649 (0.00583)	5.466020 (0.00022)
	Volume	Price	−0.004371 (−1.9566)	7.03E-05 (0.04386)	0.003132 (1.93465)	0.003132 (1.95626)	3.828360 (0.050469)	2.992141 (0.02974)	3.317081 (0.01012)
Wheat	Price	Volume	−0.005998 (−3.7838)	−0.023995 (−0.1232)	0.259590 (1.26386)	0.101044 (0.52654)	14.31724 (0.000157)	0.720722 (0.53951)	4.338591 (0.001677)
	Volume	Price	−0.016703 (−3.9304)	0.000964 (0.68592)	0.002813 (1.99197)	0.002092 (1.48737)	15.44748 (0.000086)	2.519165 (0.05627)	6.135502 (0.000064)
Oats	Price	Volume	−0.003876 (−3.0144)	−0.119789 (−1.2957)	0.019810 (0.20248)	0.028703 (0.31518)	9.086771 (0.002592)	0.852794 (0.46493)	2.702998 (0.028919)
	Volume	Price	−0.017363 (−4.0187)	−0.005668 (−1.9134)	0.003842 (1.28042)	0.004212 (1.41967)	16.14965 (0.00006)	2.353265 (0.07020)	5.992997 (0.000084)
Soybeans	Price	Volume	−0.006625 (−3.7615)	−0.542397 (−1.0867)	0.585619 (1.07938)	0.791203 (1.60232)	14.14859 (0.000172)	2.109241 (0.09698)	5.027534 (0.000486)
	Volume	Price	−0.006004 (−2.0264)	0.000919 (1.69036)	0.001901 (3.49267)	0.001030 (1.89013	4.106199 (0.042799)	6.424873 (0.00025)	6.21101 (0.000056)
Soybean meal	Price	Volume	−0.005343 (−3.4003)	−0.091127 (−0.6555)	0.377531 (2.56817)	0.210950 (1.53121)	11.56196 (0.000680)	3.573351 (0.01342)	5.742636 (0.000132)
	Volume	Price	−0.009943 (−2.8275)	0.001770 (0.90888)	0.001228 (0.63063)	0.000861 (0.44276)	7.994982 (0.004716)	0.496905 (0.68444)	2.494907 (0.040958)

TABLE V (Continued)

Error-Correction Model (FCM) for Testing for Long-Term and Short-Term Relationship for Price and Volume of Agricultural Futures Contracts

	Dependent Variable	*Independent Variable*	a_1, β_1 (*t stat*)	c_1, ϕ_1 (*t stat*)	c_2, ϕ_2 (*t stat*)	c_3, ϕ_3 (*t stat*)	H_0: *No Relationship* F stat Probability* *No LT Impact*	*No ST Impact*	*No LT or ST Impact*
Soybean oil	Price	Volume	−0.003877 (−2.8314)	0.010730 (0.57792)	0.031233 (1.57920)	0.014418 (0.78050)	8.016719 (0.004660)	0.841103 (0.47123)	2.859816 (0.022186)
	Volume	Price	−0.006460 (−2.2724)	0.007828 (0.53685)	0.002989 (0.20393)	0.012188 (0.83524)	5.163859 (0.023120)	0.354977 (0.78555)	1.639376 (0.161457)

Notes: The model is

$$Y_t - Y_{t-1} = \beta_1 \hat{Z}_{t-1} + \sum_{i-1}^{T} \phi_i (X_{t-i} - X_{t-i-1}) + \sum_{j=1}^{T} \theta_j (Y_{t-j} + Y_{t-j-1}) + \varepsilon_t \; X_t - X_{t-1} = a_1 \hat{Z}_{t-1} + \sum_{i=1}^{T} c_i (Y_{t-i} - Y_{t-i-1}) + \sum_{j=1}^{T} d_j (X_{t-j} + X_{t-j-1}) + \varepsilon_t$$

The null hypotheses are

No long-run relationship from volume to price: $\mathbf{H}_0$: $a_i = 0$
No long-run relationship from price to volume: $\mathbf{H}_0$: $\beta_i = 0$
No short-run relationship from volume to price: $\mathbf{H}_0$: $c_i = 0$
No short-run relationship from price to volume: $\mathbf{H}_0$: $\phi_i = 0$

*The null hypotheses are tested with a Wald test. Since the sample is very large, the critical values of an *F* distribution are used. The corresponding probabilities for the F statistics are reported in the table below the *F* value.

TABLE VI

Determinants of Trading Volume

Commodity	α_0, *t stat*	α_1, *t stat*	β, *t stat*	γ, *t stat*	R^2, *F stat*
Corn	−0.071519*	5.09*E*-06	−0.000779	0.028986*	0.037966
	(−5.747092)	(0.977801)	(−0.442677)	(11.90414)	(47.93624)
Wheat	−0.135956*	−1.70*E*-06	−0.001106	0.042548*	0.087619
	(−9.099619)	(−0.172827)	(−0.730850)	(18.61862)	(116.6483)
Oats	−0.102397*	7.86*E*-06	−0.003675	0.043561*	0.026137
	(−4.633792)	(0.846691)	(−1.125881)	(9.759891)	(32.59998)
Soybeans	−0.068124*	8.01*E*-06	−0.000613	0.009182*	0.034222
	(−5.374561)	(1.500056)	(−0.981719)	(11.14503)	(43.04184)
Soybean meal	−0.069666*	6.16*E*-06	−0.001114	0.032646*	0.035350
	(−5.225460)	(1.085940)	(−0.517445)	(11.51866)	(44.51247)
Soybean oil	−0.080254*	5.97*E*-06	−0.016591	0.279555*	0.042921
	(−6.089857)	(1.081941)	(−1.039628)	(12.73598)	(54.47215)

Notes: The model is

$$\Delta V_t = \alpha_0 + \alpha_1 t + \beta(\Delta P_t) + \gamma|\Delta P_t| + \varepsilon_t$$

where
ΔV_t = change in futures trading volume
t = time trend
ΔP_t = change in price
$|\Delta P_t|$ = absolute change in price (price volatility)
The *t* statistics and *F* statistics are given in parentheses.
*Significant at the 5% confidence level.

of the independent variables in the ECM, and the F statistic refers to the Wald test for causality. As described in Lutkepohl (1981, Chapter 3). This study test whether any subset of variables have zero coefficients and might thus lead to rejection of the causality.

Table V also illustrates the existence of short-term impact between price and volume for corn, soybeans, and soybean meal, but weak impact (statistically insignificant) for wheat, oats, and soybean oil. This short-term relationship is particularly strong in both directions (from price to volume and volume to price) for corn, soybean, and soybean meal. In general, the direction of causality is stronger from price to trading volume, suggesting for all six of the agricultural commodities that price tends to lead trading volume in the short run.

The third testable hypothesis postulates that changes in trading volume over time depends on three factors: time trend, price, and volatility of price, as indicated by eqs. (4) and (5). Table VI shows that only the volatility of price has a statistically significant impact on trading volume. Finally, Table VII presents the results of the fourth testable hypothesis suggested in eqs. (6) and (7) that is, the volatility of trading volume as a

TABLE VII

Volatility of Trading Volume as a Function of Price Volatility

Commodity	*α, t stat*	*δ, t stat*	*R^2, F stat*
Corn	0.229860* (48.48618)	0.015294* (10.06300)	0.027023 (101.2639)
Wheat	0.299558* (46.48788)	0.006986* (4.691702)	0.006001 (22.01207)
Oats	0.424524* (49.56369)	0.016529* (5.766557)	0.009038 (33.25318)
Soybeans	0.237279* (49.39436)	0.003917* (7.329965)*	0.014522 (53.72839)
Soybean meal	0.254984* (50.88843)	0.014905* (8.223298)	0.018209 (67.62262)
Soybean oil	0.259160* (51.02458)	0.075420* (5.388397)	0.007901 (29.03482)

Notes: The model is

$$|\Delta V_t| = \alpha + \delta|\Delta P_t| + \varepsilon_t$$

where
$|\Delta V_t|$ = Absolute change in futures trading volume (volume's volatility)
$|\Delta P_t|$ = Absolute change in price (price volatility)
The *t* statistics and F statistics are given in parentheses.
*Significant at the 5% confidence level.

function of price volatility. Table VII shows that price volatility significantly impacts volume's volatility.

6. SUMMARY AND CONCLUSIONS

This article investigates several hypotheses about the time series properties of price and trading volume, the short-term and long-term relationships between price and trading volume, and the determinants of trading volume. The data correspond to daily settlement prices and trading volume covering the time period, January 1981–September 1995, for six agricultural commodity futures contracts: corn, wheat, oats, soybeans, soybean meal, and soybean oil.

It is found that the time series of price and trading volume are nonstationary in levels but stationary in the first differences; that is, they are integrated of order 1, I(1). Because the two variables are cointegrated, there is causality in the Granger sense between price and volume of trade at least in one direction. Thus, price and trading volume are interrelated in the long run and in the short run. Cointegration, the direction of causality, and the error correction methodology suggest that trading volume tends to adjust to price in the long run and that price tends to lead trading volume in the short run. The results also indicate that price volatility is a determinant of both trading volume and volatility of trading volume.

The theoretical contribution of the article can be summarized as follows: The article develops a dynamic model relating price and volume. The model allows both price and volume to be random variables with arbitrary probability distributions that can change over time. The model postulates that price follows a continuous-time random walk with a trend known as an Itô's process. The model uses stochastic calculus to derive the result that trading volume also follows a stochastic equation of the Itô type. If volume follows an Itô process, one can compute its first two moments. The model is completed by estimating the expected volume and its volatility. The model suggests that volume is impacted by price and price volatility, and that volume's volatility is proportional to price volatility. The empirical results confirm most of the postulated hypotheses: (i) price and trading volume follow random walks and they are integrated of order 1; (ii) price and trading volume are cointegrated in the long run; (iii) the third hypothesis is confirmed only for price variability; that is, trading volume is a function of price variability; (iv) volatility of trading volume is a function of price variability.

The article finds that price and volume are cointegrated, and that this long-run relationship is stronger from price to volume. Also, this article reports bidirectional causality between price and volume and establishes the clear importance of a long-run relationship, rather than short run, between price and volume from the error correction methodology. The finding that price variability is a determinant of volume confirms previous results of Cornell (1981), Garcia et al (1986), and others. In addition, a new result is found that price variability has an impact on volume's variability.

The long-run and short-run relationships between price and volume implied by the tests of cointegration and error correction highlight the relevance of volume and offer support to technical analysis. Recall that unlike the efficient market hypothesis, which ignores trading volume, technical analysis has long emphasized the significance of volume. These results also suggest that academicians should consider the role of technical analysis in future research.

The results reported in this article have implications for speculators and hedgers. In effect, Rutledge (1979) indicates that changes in daily trading volume are a measure of variations in speculation because speculative transactions comprise most of daily trading volume. The bidirectional causality reported in this article suggests that speculators should pay attention not only to price changes, but also to changes in volume. On the other hand, the long-run underlying relationship between price and volume found in this article should be of more interest to hedgers, who hold their position in the futures markets much longer than speculators.

For example, consider a representative hedger who follows the standard methodology of computing a hedge ratio with the use of the Ederington (1979) approach. Suppose that volume is low and not very responsive to price volatility. This suggests that the market is rather illiquid, with a large bid–ask spread leading to higher volatility and affecting the size of the hedge ratio. Knowing that price volatility causes a similar change in volume has informational value. The hedger can count on volume responsiveness due to price changes. In other words, liquidity is present when price and volume are interrelated. Furthermore, such liquidity could, more often than not, reduce further price volatility and possibly decrease potential losses from ineffective hedges. Obviously, this topic requires further analysis.

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